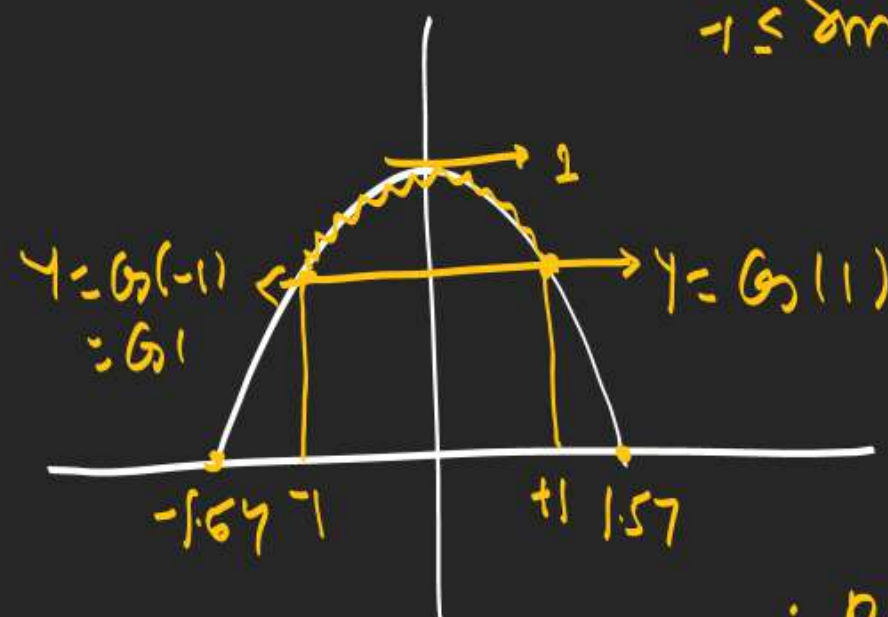


fog's Range

$$f(g(x)) = fog(x)$$

1) $y = G(\sin x)$'s Range?

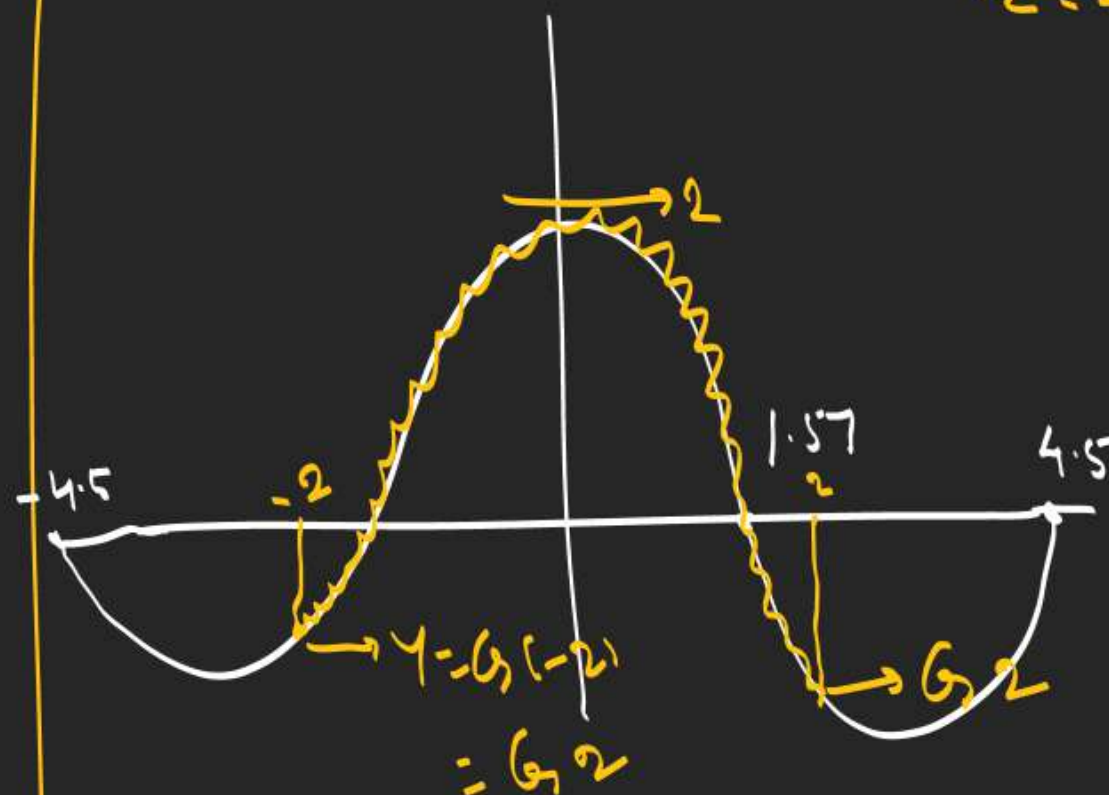
$$-1 \leq \sin x \leq 1$$



$$\therefore R_f \in [G(-1), G(1)]$$

(2) $y = G(2 \sin x)$'s Range?

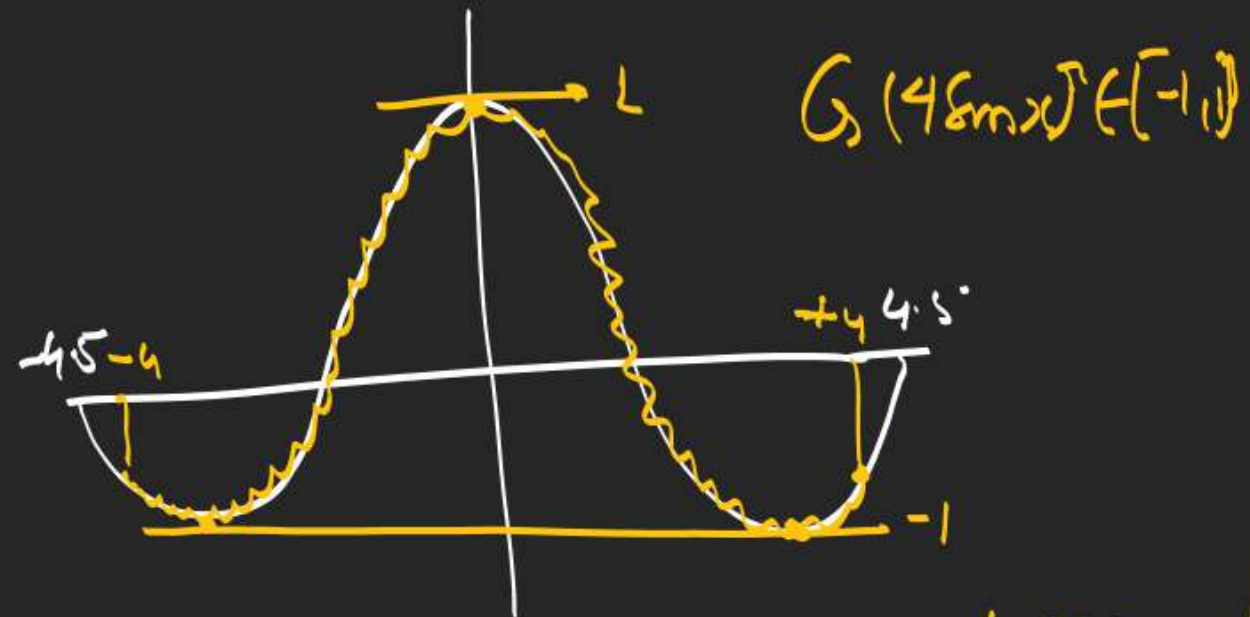
$$-2 \leq 2 \sin x \leq 2$$



$$\therefore R_f \in [G(-2), G(2)]$$

$$(3) Y = \lfloor G \rfloor (4 \sin x)$$

$$-4 \leq 4 \sin x \leq 4$$



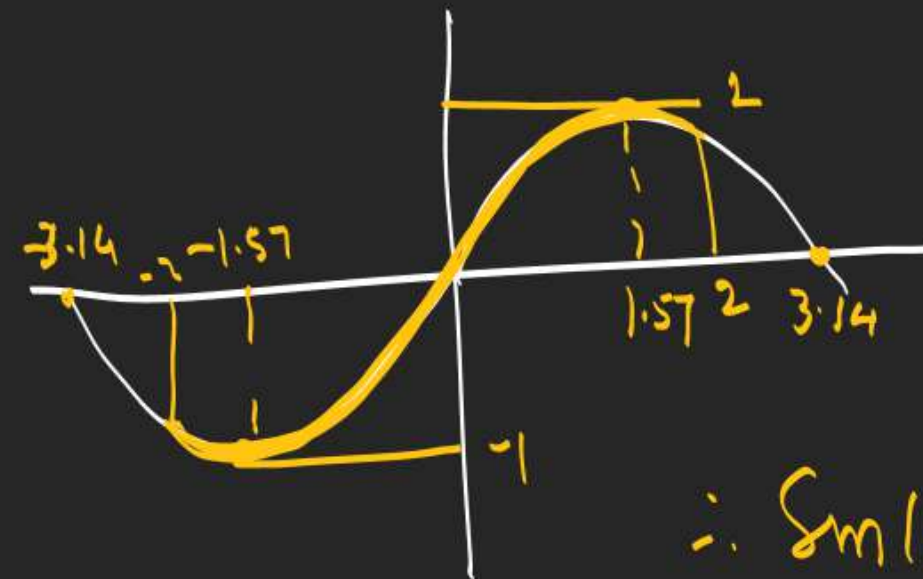
$$\frac{1.57}{\times 3}$$

Non Monotonic $\rightarrow$ Sometime $\uparrow$ Sometime $\downarrow$

$$\textcircled{1} Y = \lfloor \sin \rfloor (2 \cos x) \in \mathbb{R}_f$$

Graph Darta

& U Chhal Dekhenge.



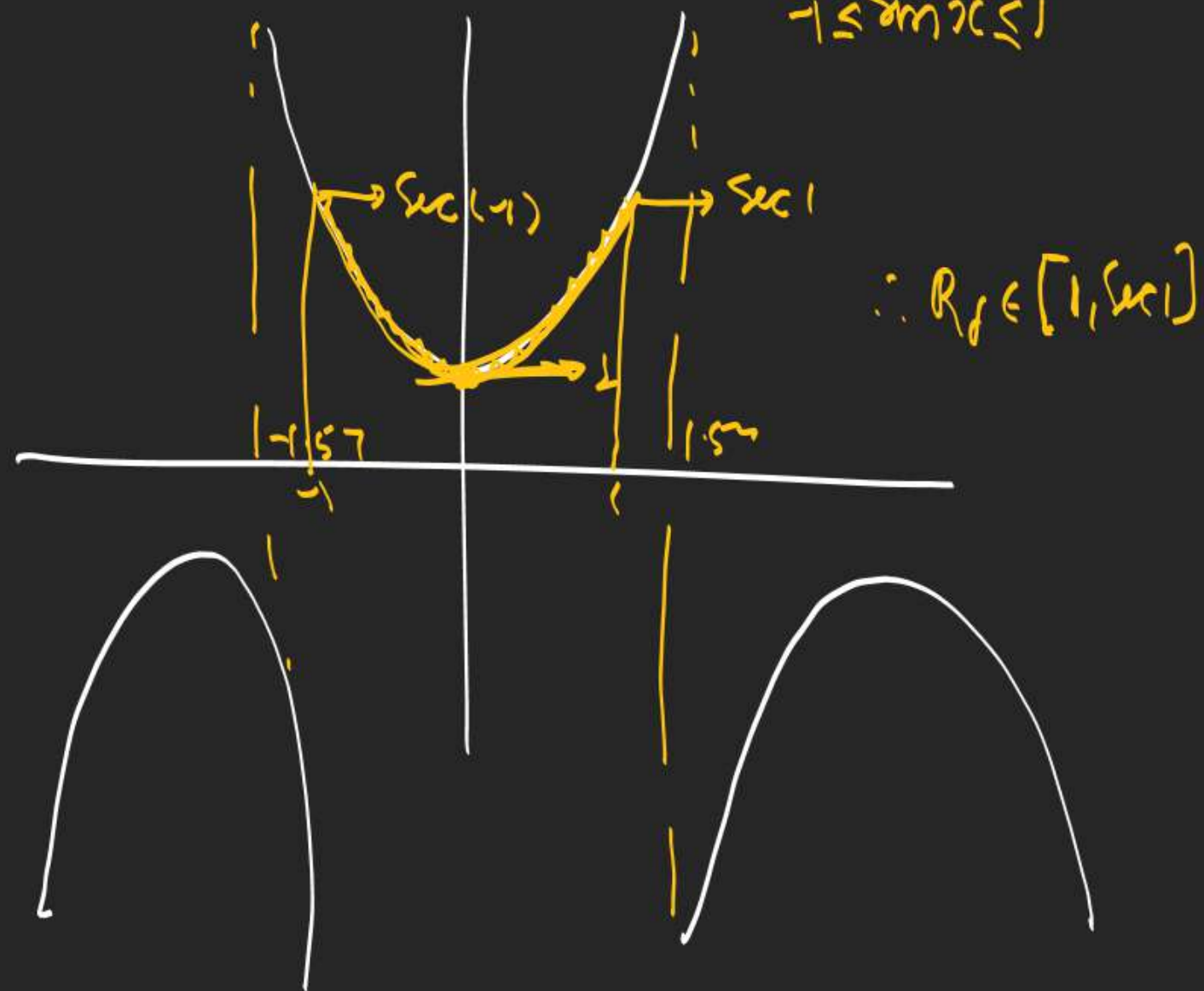
$$-2 \leq 2 \cos x \leq 2$$

$$\therefore \sin(2 \cos x) \in [-1, 1]$$

Q $y = \boxed{\text{Sec}}(\sin x)$'s Range?

Non Monotonic

$$-1 \leq \sin x \leq 1$$



Q

$y = [\sin x]$'s R_f ?

$$-1 \leq \sin x \leq 1$$

$$[-1] \leq [\sin x] \leq [1]$$

$$-1 \leq [\sin x] \leq 1$$

Known as
Discrete R_f

Integer Data!!

$$\therefore [\sin x] = \{-1, 0, 1\} = y$$

Q $y = [-\sin^{-1} x]$'s R_f ?

$$-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$$



$$[-1.57] \leq [-\sin^{-1} x] \leq [1.57]$$

$$-2 \leq [-\sin^{-1} x] \leq 1$$

$$[-\sin^{-1} x] = \{-2, -1, 0, 1\} \in R_f$$

$$y = a \sin x + b \cos x \text{ is } R_+$$

$$a \sin x + b \cos x \in [-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}]$$

$$Q \quad y = \underline{3 \sin x - 4 \cos x} + 10 \text{ is } R_+?$$

$$[-\sqrt{3^2+4^2}, \sqrt{3^2+4^2}] + 10$$

$$[-5, 5] + 10$$

$$y \in [5, 15]$$

$$Q \quad y = \sin x - \cos x \text{ is } R_+$$

$$[-\sqrt{1^2+1^2}, \sqrt{1^2+1^2}]$$

$$[-\sqrt{2}, \sqrt{2}]$$

$$Q \quad y = \log_2 \left[\frac{3 \sin x - 4 \cos x + 15}{10} \right] \text{ is } R_+?$$

R. f. Andar Se Bahar
Ki Aur.

$$-\sqrt{3^2+4^2} \leq 3 \sin x - 4 \cos x \leq \sqrt{3^2+4^2}$$

$$-5 \leq 3 \sin x - 4 \cos x \leq 5$$

$$10 \leq 3 \sin x - 4 \cos x + 15 \leq 20$$

$$1 \leq \frac{3 \sin x - 4 \cos x + 15}{10} \leq 2$$

$$\frac{3 \sin x - 4 \cos x + 15}{10} \in [1, 2]$$

$$\boxed{\log_2} \left[\frac{3 \sin x - 4 \cos x + 15}{10} \right] \in [\log_2 1, \log_2 2]$$

$$y \in [0, 1]$$



$$Q \ y = \frac{17 + 5 \sin x + 12 \cos x}{17 - 5 \sin x - 12 \cos x}, \text{ is } R_f?$$

$$y = \frac{17 + (5 \sin x + 12 \cos x)}{17 - (5 \sin x + 12 \cos x)} = \frac{17 + p}{17 - p}$$

$$p = 5 \sin x + 12 \cos x$$

$$p \in [-\sqrt{5^2 + 12^2}, \sqrt{5^2 + 12^2}]$$

$$[-13, 13]$$

$$\begin{array}{l} \text{Max} \rightarrow \frac{17 + 13}{17 - 13} = \frac{30}{4} \quad \frac{17 + 12}{17 - 12} = \frac{29}{5} \\ \text{Min} \rightarrow \frac{17 - 13}{17 - (-13)} = \frac{4}{30} \quad \frac{17 - 12}{17 - (-12)} = \frac{5}{29} \end{array}$$

$$\therefore \frac{17 + p}{17 - p} \in \left[\frac{2}{15}, \frac{15}{2} \right]$$

$$Q \ y = \log_{\frac{1}{3}} \left[\frac{5 \sin x + 12 \cos x + 14}{9} \right]$$

$\theta = \text{Argument Same}$

$$5 \sin x + 12 \cos x \in [-13, 13]$$

$$5 \sin x + 12 \cos x + 14 \in [1, 27]$$

$$\frac{5 \sin x + 12 \cos x + 14}{9} \in \left[\frac{1}{9}, 3 \right]$$

$$\log_{\frac{1}{3}} \left[\frac{5 \sin x + 12 \cos x + 14}{9} \right] \in \left[\log_{\frac{1}{3}} 3, \log_{\frac{1}{3}} \frac{1}{9} \right]$$

$$y \in [-1, 2]$$

$$Q \ y = \sin \left(x + \frac{\pi}{3} \right) + 3 \cos \left(x - \frac{\pi}{3} \right) \quad R_+?$$

$$y = \sin x \cdot \cos \frac{\pi}{3} + \cos x \cdot \sin \frac{\pi}{3} + 3 \left[\cos x \cdot \cos \frac{\pi}{3} + \sin x \cdot \sin \frac{\pi}{3} \right]$$

$$= \frac{1}{2} \sin x + \frac{\sqrt{3}}{2} \cos x + \frac{3}{2} \cos x + \frac{3\sqrt{3}}{2} \sin x$$

$$y = \underbrace{\frac{3\sqrt{3}+1}{2}}_a \sin x + \underbrace{\frac{\sqrt{3}+3}{2}}_b \cos x$$

$$\in \left[-\sqrt{\left(\frac{3\sqrt{3}+1}{2} \right)^2 + \left(\frac{\sqrt{3}+3}{2} \right)^2}, \sqrt{\left(\frac{3\sqrt{3}+1}{2} \right)^2 + \left(\frac{\sqrt{3}+3}{2} \right)^2} \right]$$

$$\in \left[-\sqrt{\frac{27+1+6\sqrt{3}+9+3+6\sqrt{3}}{4}}, \sqrt{\frac{27+1+6\sqrt{3}+9+3+6\sqrt{3}}{4}} \right]$$

$$y \in \left[-\sqrt{10+3\sqrt{3}}, \sqrt{10+3\sqrt{3}} \right]$$

When Argument are different.

Q $y = \cos 2x + 3 \sin x$ $\mathbb{R}_+$?

$$y = 1 - 2 \sin^2 x + 3 \sin x$$

$$= -[2 \sin^2 x - 3 \sin x - 1]$$

$$= -2 \left[\sin^2 x - \frac{3}{2} \sin x - \frac{1}{2} \right]$$

$$= -2 \left[\left(\sin x - \frac{3}{4} \right)^2 - \left(\frac{3}{4} \right)^2 - \frac{1}{2} \right]$$

$$= -2 \left[\left(\sin x - \frac{3}{4} \right)^2 - \frac{17}{16} \right]$$

$$\begin{array}{cccc} \left(\sin x - \frac{3}{4} \right)^2 & & & \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \left(\frac{3}{4} - \frac{3}{4} \right)^2 & \left(0 - \frac{3}{4} \right)^2 & \left(-1 - \frac{3}{4} \right)^2 & \left(1 - \frac{3}{4} \right)^2 \\ 0 & \frac{9}{16} & \frac{49}{16} & \frac{1}{16} \end{array}$$

$$0 \leq \left(\sin x - \frac{3}{4} \right)^2 \leq \frac{49}{16}$$

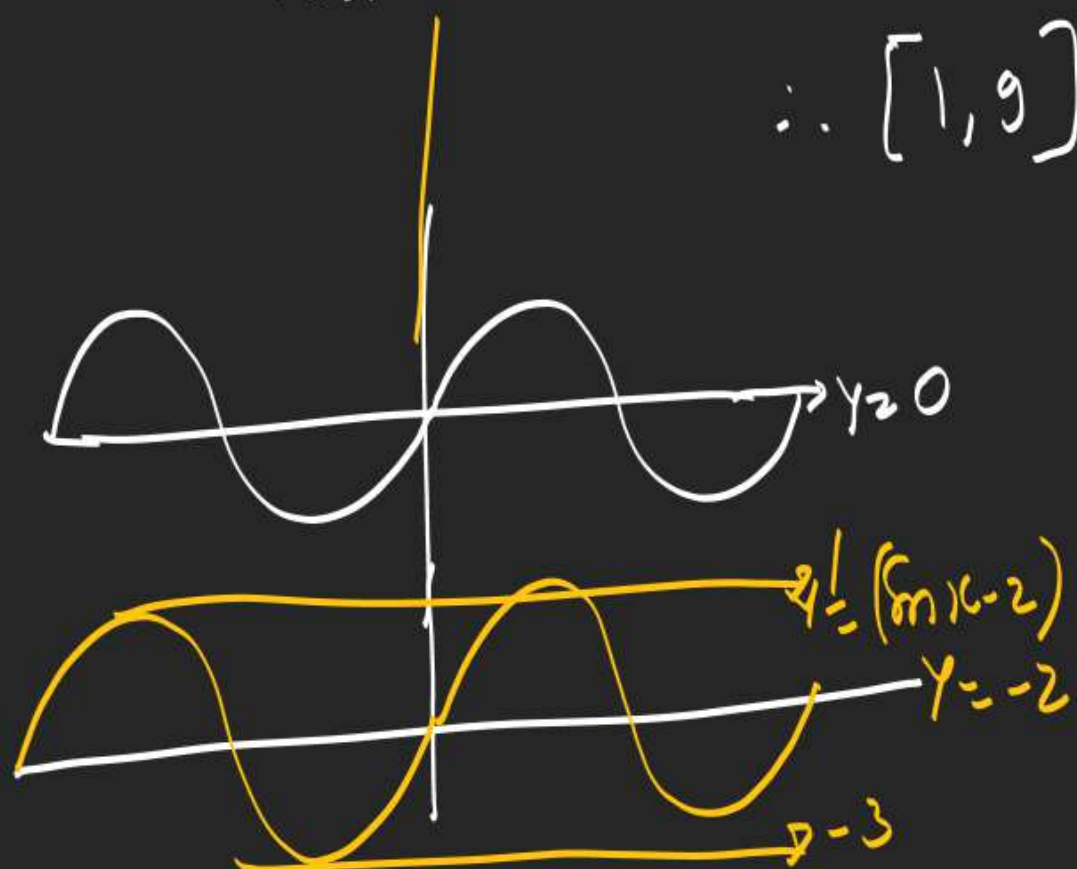
$$-\frac{17}{16} \leq \left(\sin x - \frac{3}{4} \right)^2 - \frac{17}{16} \leq \frac{32}{16} \quad \therefore y \in \left[-4, \frac{17}{8} \right]$$

$$\frac{17}{8} \geq -2 \left[\left(\sin x - \frac{3}{4} \right)^2 - \frac{17}{16} \right] \geq -4$$

Q $y = (\sin x - 2)^2 \mathbb{R}_+?$

$(2-2)^2$ (X) Not PSbl.	$(0-2)^2$ 4	$(-1-2)^2$ 9	$(1-2)^2$ 1
----------------------------------	----------------	-----------------	----------------

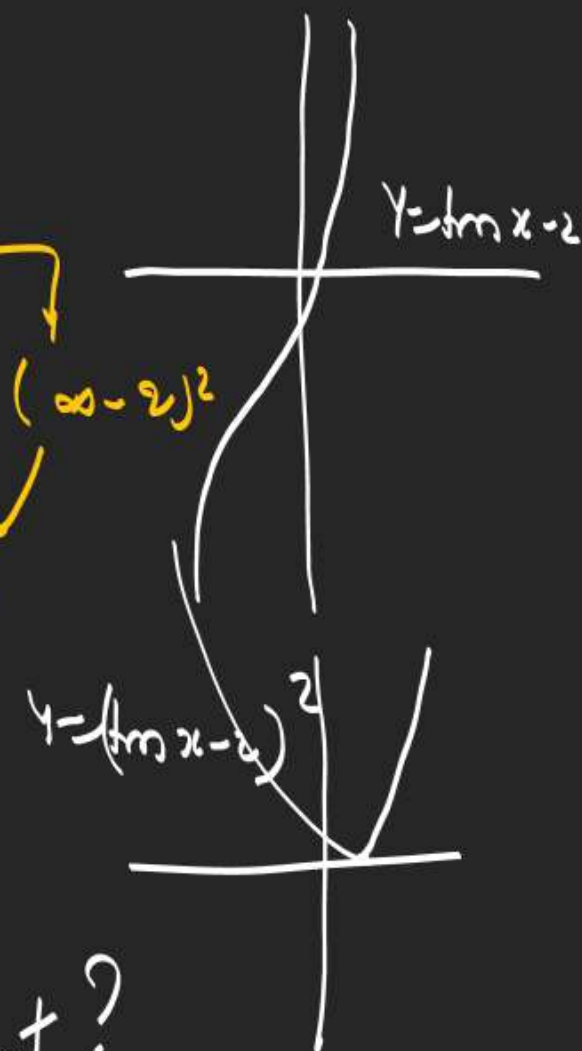
$\therefore [1, 9]$



Q $y = (\ln x - 2)^2$

$(2-2)^2$ 0 PSbl	$(-\infty-2)^2$ ∞	$(\infty-2)^2$ ∞
------------------------	-----------------------------	----------------------------

$y = [0, \infty)$



Q $y = 6x^2 - 46x + 13 \mathbb{R}_+?$

$$= (6x^2 - 46x + 4) + 9$$

$$= (6x - 2)^2 + 9$$

$$1 \leq (6x - 2)^2 \leq 9$$

$$10 \leq (6x - 2)^2 + 9 \leq 18 \therefore y \in [10, 18]$$

Reciprocal fns R+

Q R+ of $y = a^2 \tan^2 \theta + b^2 \cot^2 \theta$

$a, b > 0, \theta \in (0, \frac{\pi}{2})$

$y = a^2 \tan^2 \theta + b^2 \cot^2 \theta + 2ab - 2ab$

$y = [a \tan \theta + b \cot \theta]^2 - 2ab$

Max $\rightarrow \infty$

Min $4ab - 2ab = 2ab$

Think!

$y = [a \tan \theta - b \cot \theta]^2 + 2ab$

$a \tan \theta = b \cot \theta$

Min $= 0 + 2ab$
 $= 2ab$

$\frac{a \tan \theta + \frac{b}{\tan \theta}}{2} \geq \sqrt{a \tan \theta \times \frac{b}{\tan \theta}}$

$a \tan \theta + b \cot \theta \geq 2\sqrt{ab}$

$(a \tan \theta + b \cot \theta)^2 \geq 4ab$ Min

$\therefore [2ab, \infty)$

M2

$\frac{a^2 \tan^2 \theta + b^2 \cot^2 \theta}{2} \geq \sqrt{a^2 \tan^2 \theta \times b^2 \cot^2 \theta}$

$a^2 \tan^2 \theta + b^2 \cot^2 \theta \geq 2ab$

Q $y = 4 \sec^2 \theta + 9 \csc^2 \theta$

$y = 4 + 4 \tan^2 \theta + 9 + 9 \cot^2 \theta$

$y = 13 + (9 \tan^2 \theta + 4 \cot^2 \theta)$

Min $= [2 \times 2 \times 3, \infty) + 13$

$[12, \infty) + 13$

$[25, \infty)$

$$Q \ y = \frac{\sin 3x}{\sin x} \text{ R}_+?$$

$$y = \frac{3\sin x - 4\sin^3 x}{\sin x}$$

$$y = 3 - 4\sin^2 x$$

J J H S J J H

$$0 \leq \sin^2 x \leq 1$$

$$0 \geq -4\sin^2 x \geq -4$$

$$3 \geq 3 - 4\sin^2 x \geq -1$$

$$\therefore y \in [-1, 3]$$

Sheets $\rightarrow$ Ex 1

$$\sin x + \sin^2 x = 1$$

At G (2012 π) DPP 1 (8)

Q1

$$\sin 7x = A \sin^7 x + B \sin^6 x + C \sin^5 x + D \sin^4 x + E \sin^3 x + F \sin^2 x + G \sin x + H$$

$A + B + C + D + E + F + G + H$ $0, 1, -1,$

$$x = \frac{\pi}{2}$$

$$Q_5 \quad y = \log \frac{(\cos^2 x + \sin^4 x + 2)}{(\sin^2 x + \cos^4 x + 2)} = ?$$

$$\frac{\sin^2 x + \cos^4 x + 2}{\cos^2 x + \sin^4 x + 2} = 1$$

$$6) \quad \textcircled{\sin \theta} = x + \frac{1}{x}$$

$\textcircled{x} \quad \begin{matrix} \geq 2 \\ \leq -2 \end{matrix}$

$$\underline{\sec \theta} = \frac{\pi}{3} = \frac{3.14}{3} \approx 1.04$$

$\textcircled{x}$

$$\tan \theta = -12.122$$

$\tan \theta \in (-\infty, \infty)$

$$11) \quad x = (2 - \sec^2 181^\circ)(2 - \sec^2 182^\circ)(2 - \sec^2 183^\circ) \dots (2 - \sec^2 269^\circ) = x = ?$$

$\sec^2 45^\circ$

$\boxed{x = 0}$

DPP 2 HW

$$3) (\sec \alpha + \tan \alpha)(\sec \beta + \tan \beta)(\sec \gamma + \tan \gamma) = \tan \alpha \cdot \tan \beta \cdot \tan \gamma.$$

$$\frac{(\sec \alpha - \tan \alpha)(\sec \beta - \tan \beta)(\sec \gamma - \tan \gamma)}{(\sec \alpha + \tan \alpha)(\sec \beta + \tan \beta)(\sec \gamma + \tan \gamma)} = \frac{1}{\tan \alpha \cdot \tan \beta \cdot \tan \gamma} = \cot \alpha \cdot \cot \beta \cdot \cot \gamma.$$

$$(4) \begin{pmatrix} \cos \theta_1 \\ -1 \end{pmatrix} + \begin{pmatrix} \cos \theta_2 \\ -1 \end{pmatrix} + \begin{pmatrix} \cos \theta_3 \\ -1 \end{pmatrix} = \begin{bmatrix} -3 \end{bmatrix} \xrightarrow{\text{Kah}} \text{Ekhihaal}$$

$$\theta_1 = \pi = \theta_2 = \theta_3$$

$$\sin \frac{\theta_1}{2} + \sin \frac{\theta_2}{2} + \sin \frac{\theta_3}{2} = \sin \frac{\pi}{2} + \sin \frac{\pi}{2} + \sin \frac{\pi}{2} = 3$$

Trigonometric Eqⁿ [More Important]

↓
More Qs. are coming.

- 1) $\sin^2 x + \cos^2 x = 1$ is an Identity Not an Eqⁿ.
 2) $\sec x \times \cos x = 1$ is an Identity Not an Eqⁿ. } Eqⁿ is satisfied by Particular values only.

- 3) $\cos x = 1$ is an Eqⁿ not an Identity → It is Trigo Eqⁿ.
 $x = \frac{\pi}{6}$ (Not satisfied)

- 4) Eqⁿ Involving $\sin$ $\cos$ $\tan$ $\cot$ $\sec$ $\csc x$ is Trigo Eqⁿ.

$$Q \sin x = \frac{\sqrt{3}}{2} = ?$$

